Problem 1 ($\leqslant$5min)

Let T (for trivial) be a polynomial with integer coefficients, and 13 distinct integer roots.
Show that $\forall x \in \mathbb{Z}, |T(x)| \geqslant 2 \times 3^{13}$ or $T(x) = 0$

Problem 2 ($\leqslant$20min)

Let $m, n \in \mathbb{N}^*$, we give $f(x) = 1 + x + x^2 + ... + x^{n-1}$
Show that $f(x)$ divides $f(x^{mn}) - n$

Problem 3 ($\leqslant$20min)

Show that there exists an infinite number of polynomials P with integer coefficients, such that:
$\forall n \in \mathbb{N}^*$ 1331 divides $P(n) + 12^n$,

Problem 4 ($\leqslant$30min)

Let $m \in \mathbb{N}$. Find a polynomial Q such that:
$\forall x \in \mathbb{Z}, Q(x) = x^m + \sum_{k=0}^{m-1} a_k x^k$ with $a_k \in \mathbb{Z}, (\forall k)$ and verifies $1995 | Q(x), (\forall x)$

Problem 5 ($\leqslant$60min)

The infinite sequence $a_0, a_1, a_2...$ of (not necessarily different) integers has the following properties:
$0 \leqslant a_i \leqslant i$ for all integers $i \geqslant 0$ and $C_k^{a_0} + C_k^{a_1} + ... + C_k^{a_k} = 2^k$ for all integers $k \geqslant 0$.
Prove that for all $N \geqslant 0, \exists i \geqslant 0$ with $a_i = N$.