mcube@tuta.io

Problem 1 ($\leqslant$15min)

Let $f(x) = \sum_{k=0}^{n>2} a_k x^k$ with $a_k \in \mathbb{R}$ $(\forall k)$
Knowing that f has exactly n real distinct roots, show that $\forall x \in \mathbb{R}$, $f'^2(x) - f(x)f''(x) > 0$

Problem 2 ($\leqslant$30min)

For $n \in \mathbb{N}^*$, let $\sigma(n)$ be the sum of all divisors of n. Show that: $\sigma(n) \leqslant n(1 + ln(n))$

Problem 3 ($\leqslant$30min)

Let P be a polynomial with real coefficients such as $\forall x \in \mathbb{R}$, $P(x) \geqslant 0$. Let $n = deg(P)$ and:
$Q = P + P' + ... + P^{(n)}$. Show that $\forall x \in \mathbb{R}$, $Q(x) \geqslant 0$

Problem 4 ($\leqslant$60min)

Calculate: $\sum_{k=0}^{p<n} (-1)^k C_n^k C_{n-k}^{p-k}$
Let D_n be the number of permutations of S_n (Set of n elements) with no fixed point, show that:
$\sum_{k=0}^{n} C_n^k D_{n-k} = n!$ (we give $D_0 = 1$)
Show that: $D_n = n!(\sum_{k=0}^{n} \frac{(-1)^k}{k!})$